# A Study on Stress Analysis of Orthotropic Composite Cylindrical Shells with a Circular or an Elliptical Cutout

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The stress analysis on orthotropic composite cylindrical shells with one circular or one elliptical cutout subjected to an axial force is carried out by using an analytical and experimental method. The composite cylindrical shell governing equation of the Donnell's type is applied to this study and all results are presented by the stress concentration factor. The stress concentration factor is defined as the ratio of the stress on the region around a cutout to the nominal stress of the shell. The stress concentration factor is classified into the circumferential stress concentration factors and the radial stress concentration factors due to the cylindrical coordinate of which the origin is the center of a cutout. The considered loading condition is only axial tension loading condition. In this study, thus, the maximum stress is induced on perpendicular region against axial direction, on the coordinate. Various cutout sizes are expressed using the radius ratio, $\theta = \pm\frac{\pi}{2}$, which is the radius of a cutout over one of the cylindrical shell. Experimental results are obtained using strain gages, which are attached around a cutout of the cylindrical shell. As the result from this study, the stress concentration around a cutout can be predicted by using the analytical method for an orthotropic composite cylindrical shell having a circular or an elliptical cutout.

**Key Words :** Cutout, Stress Concentration Factor (SCF), Radius Ratio ($\rho$)

## 1. Introduction

The stress distribution in a flat plate with a circular cutout has been investigated for the first time by Kirsch (1898). His results pertaining to an infinite plate loaded in unidirectional tension are well-known. The maximum stress on the flat plate having a circular cutout amounts to three times the maximum stress that would be found in a solid plate. This factor 3 is known as the stress concentration factor in the plate. Many solutions for other problems of this type have been obtained, e.g. different loading conditions, plates that are not infinite such as strips, and the cases of non-circular cutouts.

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It must be expected that in the case of a cylindrical shell weakened by a circular cutout these results will be influenced by the curvature. The first one who attacked this problem was Lur'e (1946). After him many researchers have studied on cutout problems and the stress distribution is predicted around the cutout smaller than 1/4 of the diameter of the shell (Lekkerkerker, 1972; Van Dyke, 1965; Eringen et al, 1964). For a large size cutout, Steele (1983, 1986) carried out the stress analysis by using Donnell's and Sanders' shell equation. And Xue et al.(1991) developed an analytical solution for the stress distribution on a cylindrical shell with a large cutout by the modified Morley's shell equation. Lee et al. (1999) studied the stress concentration and the stress distribution near a cutout of the cylindrical shells with a reinforced cutout under an axial compression. Lee et al.(1997) researched the stress contour over both a reinforcement and main shell of the reinforced radial nozzle on the spherical shell. Chen et al.(2002) studied the elastic analysis of doubly periodic circular holes in infinite plane. Lee et al.(2002) also examined the circular cylinder with many cylindrical holes in axial direction under thermal loading. All these studies are valid for an isotropic material cylindrical shell. But there are a few researches for the stress distribution on the composite shell with a cutout. Mukoed (1970) obtained the analytical solutions for the orthotropic cylindrical shells with a circular cutout using the Hankel-Kylov function. Researches for a composite material with any cutout were usually applied to a flat plate.

In this paper, the stress analysis for the orthotropic composite cylindrical shells with a circular or an elliptical cutout subjected to an axial force is carried out using an analytical and an experimental method. In the analytical method, the composite shell governing equation of Donnell's type is used and all results are indicated by the stress concentration factor that is defined as the ratio of the stress around a cutout to a nominal stress on the cylindrical shell. The various cutout sizes are defined as radius ratio, the radius of a cutout to one of the cylindrical shell. Experimental results are calculated using the strain gages which are attached on both the inner surface and the outer surface of a cylindrical shell.

## 2. Formulation

To analyze the cylindrical shell having a cutout, many coordinate systems and the relation ship among the systems must be understood. Fig. 1 shows considered coordinate systems for the cutout problem. $R$, $r_0$ and $t$ represent the radius of a cylindrical shell, the radius of a cutout and the thickness of a cylindrical shell, respectively. To determine the cutout problem, 4 coordinate systems are considered; a global Cartesian coordinate system $(x, y, z)$, a developed Cartesian coordinate system $(\xi, \varphi, z)$, a developed cylindrical coordinate system $(\alpha, \beta, z)$ and a projected cylindrical coordinate $(\rho, \theta, z)$. In the projected cylindrical coordinate $\rho$ denotes radius ratio and is defined as $r/R$.

The Donnell's governing equation for the cylindrical shells of an isotropic material is well known as one of the governing equations. Mukoed (1970) studied stress state in the vicinity of the circular hole in an orthotropic composite cylindrical shells. In his study, the governing

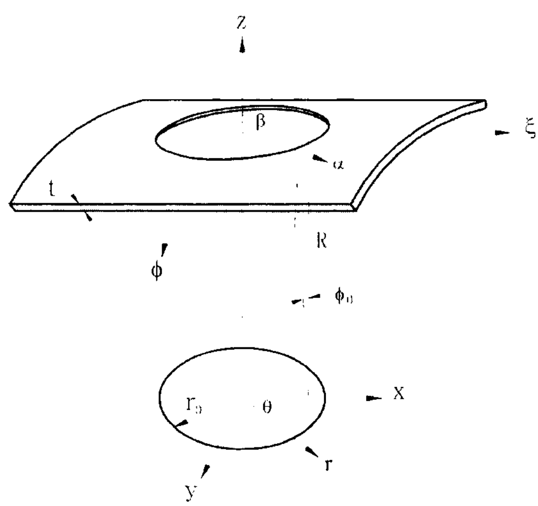


**Fig. 1** Coordinate systems for the cylindrical shell with a cutout $(\rho=r/R)$

equation of Donnell's type on orthotropic cylindrical shells through complex form is obtained as following

$$\left[\left(\frac{\partial^2}{\partial\xi^2}+\lambda\frac{\partial^2}{\partial\phi^2}\right)\nabla^2-4\mu^2 i\frac{\partial^2}{\partial\xi^2}\right]\chi=0 \tag{1}$$

where, the orthotropic parameter $\lambda$ and the displacement-stress function $\chi$ are defined

$$\lambda=\frac{E_\phi}{E_\xi} \tag{2a}$$

$$\lambda=w+i\frac{2}{t^2}\sqrt{\frac{3(1-\nu_{\xi\phi}\nu_{\phi\xi})}{E_\xi E_\phi}}\,\Phi \tag{2b}$$

$$\mu=i\frac{r_0}{2\sqrt{Rt}}[12\lambda(1-\nu_{\xi\phi}\nu_{\phi\xi})]^{1/4} \tag{2c}$$

Eq. (1) can be written as following

$$\left(\nabla^4-4\mu^2 i\frac{\partial^2}{\partial\xi^2}\right)\chi=\gamma\frac{\partial^2}{\partial\phi^2}\nabla^2\chi \tag{3a}$$

$$\gamma=1-\lambda \tag{3b}$$

Let us assume that $|\gamma|<1$ and the function $\chi$ can be represented in the form of a series expansion with respect to $\gamma$

$$\chi=\chi^0+\gamma\chi^1+\gamma^2\chi^2+\cdots \tag{4}$$

Using this equation we can obtain a homogeneous equation and a particular equation from the governing equation.

$$\left(\nabla^4-4\mu^2 i\frac{\partial^2}{\partial\xi^2}\right)\chi_0=0 \tag{5a}$$

$$\left(\nabla^2-4\mu^2 i\frac{\partial^2}{\partial\xi^2}\right)\chi_j=\frac{\partial^2}{\partial\phi^2}\nabla^2\phi_{j-1} \tag{5b}$$

where, $j=1, 2, \cdots$

The right side of both equations is similar to one of the governing equation for isotropic cylindrical shell. The particular equation (5b) is involved by the orthotropic characteristics of material. But the particular solution may be ignored because the homogeneous solution is more dominant to the stress prediction for this problem than the particular solution.

The displacement-stress function is derived using a Bessel and Hankel function.

$$\chi_0=\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}(-1)^k C_n F_{kn}\cos 2k\beta \tag{6}$$

$$F_{kn}=\begin{cases} J_{-n}(\sqrt{-i}\,\mu\alpha)H_n(\eta\alpha) & (k=0)\\ [J_{2k-n}(\sqrt{-i}\,\mu\alpha)+J_{-2k-n}(\sqrt{-i}\,u\alpha)]H_n(\eta\alpha) & (k>0)\end{cases} \tag{7}$$

where the coefficient is

$$\eta=\mu\sqrt{-i} \tag{8}$$

$C_n$ is the unknown complex constant which can be obtained applying the free boundary condition along the cutout edge.

The cutout size for a circular and an elliptical cutout is indicated by the radius ratio ;

$$\rho_0=\frac{r_0}{R} \quad \text{: circular cutout} \tag{9a}$$

$$\rho_0=\frac{ab}{R\sqrt{a^2\sin^2\theta+b^2\cos^2\theta}} \quad \text{: elliptical cutout} \tag{9b}$$

where $a$ and $b$ for an elliptical cutout are a major and a minor radius of the cutout, respectively. And the normal direction coordinate, $z$, is described the function of circumferential direction coordinate, $\theta$, along the boundary curve of a cutout, $\Gamma$, in the coordinates $(\rho, \theta, z)$.

$$\begin{cases}\rho_\Gamma=\rho_0\\ z_\Gamma=R\sqrt{1-\rho_0^2\sin^2\theta}\end{cases} \tag{10}$$

That is transformed on the coordinates $(\alpha, \beta)$

$$\begin{cases}\alpha_\Gamma=[\rho_0^2\cos^2\theta+\{\sin^{-1}(\rho_0\sin\theta)\}^2]^{1/2}\\ \beta_\Gamma=\sin^{-1}[\{\sin^{-1}(\rho_0\sin\theta)\}\\ \qquad\{\rho_0^2\cos^2\theta+(\sin^{-1}(\rho_0\sin\theta))^2\}^{-1/2}]\end{cases} \tag{11}$$

## 3. Experiment

Uniform axial tension is considered as the loading condition on the composite cylindrical shell. The used composite material is GFRP plain weave fabric composite and the laminated sequence is $[0_2/\pm45_2/90_2]_s$. A plain weave fabric composite is generally used due to some manufactured convenience and the sequence is one of general cases. The material properties are presented in Table 1.

Geometric dimensions of the cylindrical shell with a elliptical cutout are following as ; radius of the cylindrical shell 107 mm, major and minor radius of the cutout 45 mm and 32 mm, thickness 2.2 mm. Fig. 2 shows the cylindrical shell having

**Table 1** Material properties of plain weave composite specimens

| Case | Unit | Value |
|---|---|---|
| Longitudinal Young's modulus ($E_1$) | GPa | 26.2 |
| Transverse Young's modulus ($E_2$) | GPa | 26.2 |
| Longitudinal shear modulus ($G_{12}$) | GPa | 4.9 |
| Poisson's ratio ($\nu_{12}$) | — | 0.12 |

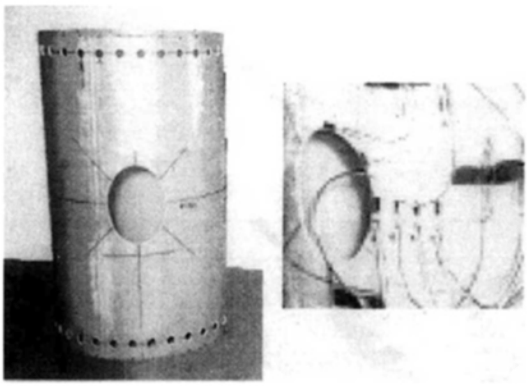

**Fig. 2** Composite cylindrical shell having an elliptical cutout and strain gages attached around the cutout edge

an elliptical cutout and strain gages attached around the cutout. Many small holes were made at both edges in axial direction of the shell in order to connect tension equipments. Strain gages were attached on $\theta=0, \frac{\pi}{4}, \frac{\pi}{2}$ and the distance between the gages in each $\theta$ direction was 13 mm. One gage is attached opposite the cutout in order to obtain the reference strain, which is used in calculating the concentration factors. Strain gages were equipped on the outer surface at the same position of the cylindrical shell and several gages were on the inner surface of the same position as the outer ones.

## 4. Review of Results

Analytical results are classified into a membrane component and a bending component, and the total stress concentration factor is obtained by combination of the two components. The total stress concentration factor on the convex region (tensile stress state) is calculated by the sum of the two components, when a structure is in a bending mode. In this study, the total SCF at $\theta=\frac{\pi}{2}$ was calculated by the difference of the two components in order to comparison with some experimental data obtained on the outer surface of the cylindrical shell. The reason is that the outer surface has a concave bending mode deflection at $\theta=\frac{\pi}{2}$ when the cylindrical shell having a cutout is under the tension load condition. So, the outer surface is in compressive stress state and the SCF obtained from experimental data can be discussed with the difference of the membrane SCF and the bending SCF, finally.

$$SCF^m=\frac{Stress\ resultant}{Nominal\ membrane\ stress\ resultant} \tag{12a}$$

$$SCF^b=\frac{6}{t}\frac{Stress\ couple}{Nominal\ membrane\ stress\ resultant} \tag{12b}$$

$$SCF=SCF^m \pm SCF^b \tag{12c}$$

All stress concentration factors obtained by analytical method equal to strain concentration factors determined by experimental method. And then comparison of analytical results with experimental results can be described easily.

Figure 3 describes experimental strain concentration factor (SCF) with radius ratio at $\theta=\frac{\pi}{2}$ on the composite cylindrical shell having an elliptical cutout to show the difference of strains on the inner and outer surface under axial loading condition. The difference is involved by bending effect around a cutout. The maximum SCF is appeared at edge of the cutout and values on the inner surface is larger than ones on outer surface in all considered region. The difference decreases steadily with increasing the radius ratio.

Figure 4 and 5 show the stress concentration factor (SCF) at $\theta=\frac{\pi}{2}$ by analytical and experimental method on a circular and elliptical cutout, respectively. The membrane stress component dominates the bending stress component in total stress. For a circular cutout problem, the total SCF is a little bit larger than the experiment SCF, while for an elliptical cutout problem, the total SCF is smaller than the experiment SCF. Comparing two figures, the maximum SCF for elliptical cutout larger than one for circular cutout.

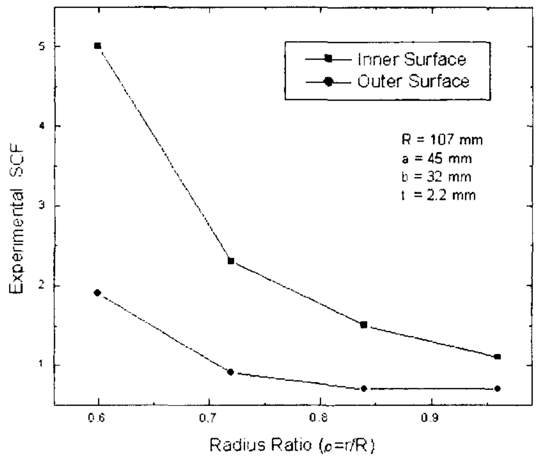


**Fig. 3** Comparison of experimental SCF on inner and outer surface

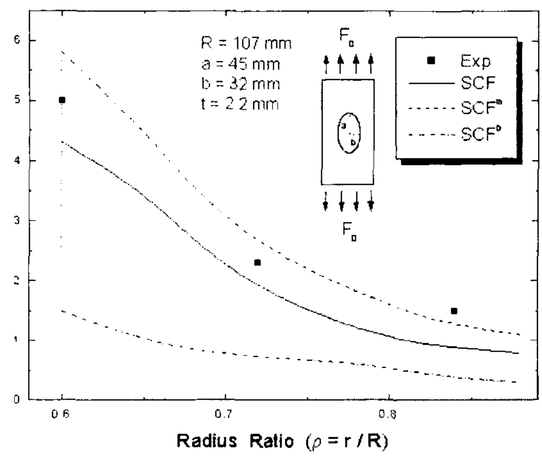


**Fig. 5** SCF on the composite cylindrical shell with an elliptical cutout

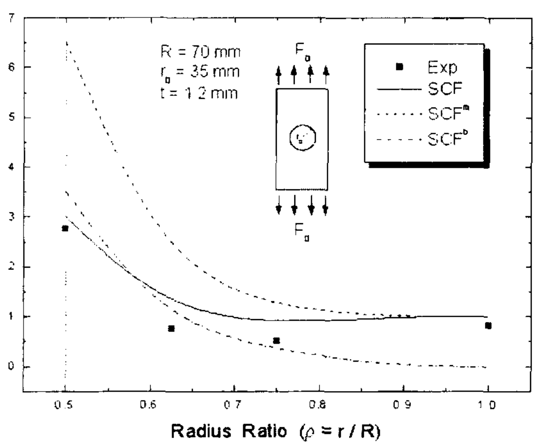


**Fig. 4** SCF on the composite cylindrical shell with a circular cutout

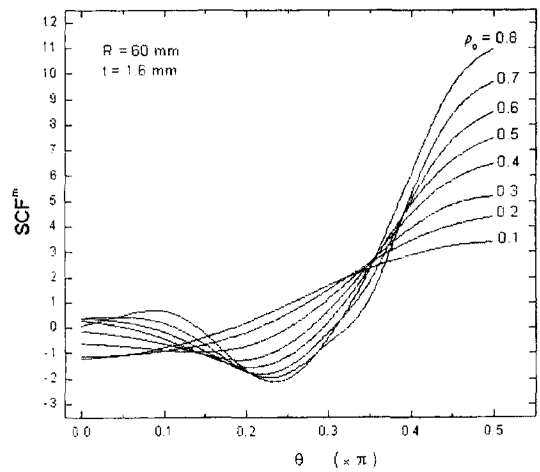


**Fig. 6** Membrane SCF on various circular cutout size

That result is not the same as well-known phenomenon. It is true that considering the same specimen in the dimensions and the same area of the cutout, SCF for the cylindrical shell with an elliptical cutout would be smaller than one for a circular cutout problem. But the results from considered problems in this paper are involved by the reason that the geometric dimensions and the cutout area of two specimens are different in each other.

Figure 6 indicates the membrane SCF along a cutout edge, which is the dominant component in the total SCF, for various circular cutout sizes. The maximum membrane SCF appears at $\theta=\frac{\pi}{2}$ and increases with increasing the cutout size. For the cutout size $\rho_0=0.1$, the membrane SCF at $\theta=0$ has minus signature and steadily increases to $\theta=\frac{\pi}{2}$. Around $\theta=\frac{\pi}{4}$, the compressive component stress shows the maximum value and the larger of a cutout size make the bigger of compression stress. Generally a cylindrical shell without any cutout under axial tension involves only the tension stress states overall the shell. But the cylindrical shell with any cutout under axial tension has various deflection shapes and so some region around the cutout may have the compressive stresses.

## 5. Conclusions

Stress analysis on the orthotropic composite cylindrical shell having one circular or an elliptical cutout is carried out by using analytical and experimental method. The composite shell governing equation of Donnell's type is used and 4 coordinate systems are applied in order to present curvature effects. The considered loading condition is axial tension condition and all results are indicated by the stress concentration factor (SCF).

(1) The maximum stress on an orthotropic composite cylindrical shell with one circular or an elliptical cutout appears at $\theta=\frac{\pi}{2}$.

(2) Due to axial tension load condition, stress near a cutout on inner surface is larger than one on outer surface of the cylindrical shell with a cutout.

(3) The membrane stress components dominate the bending components and the total stresses are obtained by the combination of these two components. The analytical results are agreement with the experimental results. And then the presented analytical method will be used to predict the stress around the cutout for an orthotropic composite cylindrical shell.

(4) The maximum membrane SCF on the edge of a cutout is proportional to the cutout size, $\rho_0$.

(5) The membrane SCF for a large cutout size is dramatically changed along cutout edge from $\theta=0$ to $\theta=\frac{\pi}{2}$, while the SCF for a small cutout size is steadily changed.

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